Chalk

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Chalk: Materials and Concepts in Mathematics Research

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1 Chalk in Hand

> Chalk in hand, his formulas expressed themselves, it seems, more easily on the board than they were able to with pen in his notebooks, for in his listeners' presence his fecund genius found again a new zeal, and a ray of joy illuminated the lines of his face when the proof he sought to render understandable struck his audience with obviousness.[1]

So recounts an admiring biographer the pedagogical exploits of Augustin-Louis Cauchy, a towering figure of early nineteenth-century mathematics. Cauchy was trained and then taught at the prestigious Ecole Polytechnique, a school for military engineers that not long before Cauchy's matriculation became one of the first to make systematic use of a new mode of advanced mathematical instruction: lessons at a blackboard. Today, chalk and blackboards are ubiquitous in mathematics education and research. Chalk figures prominently in the imaginations and daily routines of most mathematicians.

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[1] Valson 1868, 1:253, our translation from the French original.

For Cauchy's biographer, there was an organic link between chalk, genius, audiences, mathematical proof, obviousness, and understanding. This link persists to this day. There is, we contend, an essential relationship between the supposedly abstract concepts and methods of advanced mathematics and the material substituents and practices that constitute them. This process operates even in the rarefied realm of mathematical research, where the pretense of dealing purely in abstract, ideal, logical entities does not liberate mathematicians from their dependence on materially circumscribed forms of representation. That this self-effacing materiality is often unnoticed (unlike the visible and controversial materiality of computerized mathematical proof analyzed by MacKenzie 2001) makes the case of research mathematics all the more important to the social study of theoretical representations. Indeed, the very appearance of scholarly mathematics as a realm apart is a social achievement of practices that produce mathematical ideas using material surrogates.

This chapter reports a series of ethnographic findings centered on the theme of chalk and blackboards as a way of illustrating the distinctive modes of inscription underlying mathematical research. Chalk, here, functions both as a metaphor and as a literal device in the construction and circulation of new concepts. We begin, after a brief review of extant literature, by describing the quotidian contexts of such work. We then explore the blackboard as a site of mathematical practice before finally expanding on its metaphorical and allusive significance in other forms of research.

Our observations have a dual character. On the one hand, we describe the supposedly distinctive realm of mathematics in a way that should appear consonant with other scholarly disciplines that one might imagine to be rather different from it. Observations that would be "old news" about other sciences or unsurprising to those acquainted with mathematical practice are nevertheless significant in a context where so few investigations of the sort we report here have been undertaken. On the other hand, we aim to account in some small way for the distinctiveness of mathematics, both as a field of study with its own characteristic objects and practices and as a domain that succeeds in appearing far more distinctive than we would suggest is the case.

In our account, the formal rigor at the heart of mathematical order becomes indissociable from the "chalk in hand" character of routine mathematical work. We call attention to the vast labor of decoding, translating, and transmaterializing official texts without which advanced mathematics could not proceed. More than that, we suggest that these putatively passive substrates of mathematical knowledge and practice instead embody constant pressures and constraints that shape mathematical research in innumerable ways.

2 Prior Accounts

This conclusion, developed through Barany's recent ethnographic study of university mathematics researchers,[2] builds on related literatures in the sociology and history of logic and the natural sciences, the history of mathematics, and the sociology of settled mathematics. Closest in methods and analytical orientation is a range of historical and ethnographic accounts of university researchers in 'thinking sciences' such as theoretical physics,[3] artificial intelligence,[4] and symbolic logic.[5] These accounts collectively demonstrate how intersubjective resources are mobilized and disputed in the production of abstract accounts of physical, social, or logical entities. Their concern for the connection between theories and their means of articulation draws from early laboratory studies that documented the artifactual achievement of circulable data and principles of scientific knowledge through the use of

[2]For a full account of the study's methods and findings, see Barany 2010. Barany observed the weekly seminar and conducted a series of interviews exploring the everyday research practices and diachronic research developments of a group of early- and mid-career mathematicians studying partial differential equations and related topics at a major British research university.

[3]Of particular note are Ochs *et al.* 1994, 1996, and 1997, Galison 1997, Merz and Knorr Cetina 1997, and Kaiser 2005.

[4]Suchman 1990 and Suchman and Trigg 1993.

[5]See Rosental 2004, 2008, and Greiffenhagen 2008.

instruments and other means of "inscription" or "rendering" that tame and transform specimens of nature.[6]

Two bodies of scholarship help us to adapt the foregoing insights to mathematics. Historians and some empirically-minded philosophers have traced the elaboration of specific mathematical theories and techniques using a variety of frameworks.[7] Sociologists, meanwhile, have described mathematical pedagogy at many levels,[8] elementary proofs and examples,[9] and (less often) advanced theorems,[10] detailing in each case the modes and means of intelligibility for already-established mathematical ideas. Some take an explicitly cognitive approach[11] and stress the mental and corporeal structures that ground mathematical thinking.

Most users of advanced mathematics, and indeed most mathematicians themselves, spend most of their time dealing with settled mathematics. This is the mathematics of teaching and of many forms of problem solving, even when these require deploying accepted results and methods in new ways, and it has generally proven amenable to social and historical analysis. Due to the obfuscations of temporal distance and conceptual difficulty, however, historians and sociologists of mathematics have struggled to account for the ongoing achievement of original knowledge in a research context, such as has been ventured for laboratory sciences. At present, those wishing to understand the core activity in most mathematicians' aspirations and self-identity must rely on accounts by mathematicians themselves or philosophically-oriented treatises on the subject.[12] While we cannot pretend to fill this lacuna, our

[6]Woolgar 1982 offers an early assessment of the literature; Lynch 1985 and Latour and Woolgar 1986 are two influential examples, treating "rendering" and "inscription" respectively; see also Lynch 1990 on the mathematical ordering of nature and Woolgar 1990 on documents in scientific practice.

[7]For example, Lakatos 1979, Bloor 1973, 1976, 1978, Mehrtens 1990, Pickering 1995, Netz 1999, Jesseph 1999, and Warwick 2003.

[8]Lave 1988, Kirshner and Whitson 1997, Greiffenhagen and Sharrock 2005.

[9]Livingston 1999, Bloor 1976, Rotman 1988, 1993, 1997.

[10]Livingston 1986, MacKenzie 2001.

[11]E.g. Netz 1999, Lakoff and Núñez 2000. See also Hutchins 1995, Mialet 1999.

[12]Prominent ones include DeMillo, Lipton, and Perlis 1979, Davis and Hersh 1981, and Thurston 1994; see also Heintz 2000 and Aschbacher 2005.

study offers a model for how such an account might proceed.

3 Mathematics in Action

On Mondays during term, members of the Analysis Group return from lunch and assemble to hear a local or invited colleague's hour-long presentation on the fruits and conundrums of his or her[13] recent and ongoing scholarship. These lectures are marked by a shared specialized vocabulary and expertise and sometimes-spirited outbursts of discussion over technical details. One gets the impression, however, that the specific mathematics of the presentation is of at best marginal interest to most of the gathered audience. Some jot notes or furrow their brows, but one is just as likely to see someone nodding off to sleep as nodding in agreement. Most audience members regard the speaker with a brand of reserved attentiveness that is easily mistaken for comprehension.

Lurking in the seminar's subtext and between the lines of multiple interviews was the open secret that mathematicians—even those in the same field, working on the same topics, or veterans of multiple mutual collaborations—tend to have comparatively little idea of what each other does.[14] Mathematics is a staggeringly fragmented discipline whose practitioners must master the art of communicating without co-understanding. Indeed, mathematicians seem persistently preoccupied with sharing their work with each other, boldly blinding themselves to the petty incomensurabilities of their studies in order to join, on scales ranging from meetings with collaborators to international congresses, in mutual mathematical activity.

Seminar performances are conditioned on a form of understanding whose pervasive presence and role in mathematical education and research stands in stark contrast to its minor role in extant accounts of mathematical proof and

[13]Though women occasionally were present at the Analysis Seminar, all of the speakers during the period of Barany's study were men.

[14]Thurston (1994, 165 *et passim*) notes something similar, and Merz and Knorr Cetina (1997, 74) identify a comparable phenomenon in theoretical physics.

cognition. Most in the seminar audience do not aim for a detailed working knowledge of the results being presented—this can take years to acquire (after which the talk would not have much to offer)—but rather comprehend the talk in the sense of *following the argument*, engaging with the talk's conceptual narrative and technical and heuristic manipulations.

This "following" mode is reflected in how both speakers and participants prepare for the seminar. Which is to say, in large part, how they do not prepare. Audience-members do not typically study for upcoming talks by looking into the speaker's topic or previous work. Seminar-goers are easily bored and prone to distraction, said one informant, adding that they rarely care in any event about the details behind the speaker's findings. Speakers indicated that their preparations, depending on the formality and importance of the occasion, ranged from "exactly four minutes" (an underestimate, but not a wholly misleading one) to a week of sporadic effort. For a chalk lecture, a single draft of highly-condensed notes suffices.

Nearly all of the speaker's words and a varying but typically large portion of what is written on the blackboard during a seminar are produced extemporaneously. Speakers are expected to produce written and oral expositions with limited reference to notes, which serve primarily to help the speaker to recall precise formulations of nuanced or complex theorems or definitions. One result of the speaker's lack of premeditation regarding inscriptions is the frequent need to adjust notations mid-lecture—notations which do not necessarily correspond to the ones used in the limited paper notes the speaker had prepared.

Talks are not, of course, pulled from thin air. Rather, they rely on mathematicians' skill, honed through years of teaching, presenting, and interacting with colleagues, of constructing an argument at the board from a collection of principles and conventions. These arguments are built out of shared rhetorical scripts and graphical representations, practiced over many years and in many settings, that govern how commonly-used ideas and methods are described and inscribed in mathematical discourse.[15] Those conventions also connect chalk-writing to speaking, so that those who make a record of the talk tend only to transcribe text from the board, making comparatively few notes from the spoken component of the presentation.

Seminars are thus conditioned on a great deal of shared training in discursive and conceptual norms. Typically, however, the speaker's and audience's expertise and interests align only superficially. As one speaker put it: "it's not clear that there's anything in the intersection of what this person's thinking of and what I know how to do." But the seminar is far from pointless. "It's a bit like a beehive," the same speaker volunteered a few days before his talk: "Collecting nectar and pollen doesn't benefit the specific bee so well, but it's important for the community."

Indeed, seminar attendance is among the chief manifestations of the Analysis Group as a community. During the lecture, speakers constitute other communities as well by framing their research in terms of recognizable problems and approaches. These larger communities, organized around particular expertise, structure the kinds of reasoning that can be implicated in a seminar's "following" activity. Researchers develop expectations about arguments, so that, as one explained, "If the argument is sort of well-established,... it can be the case that people know where it's going to break if it's going to break." Specialisms also supply canonical terms and arguments, dictating what claims can be made (and how) without further justification.

Specialism-specific ways of describing objects and rendering them on blackboards and other media are enculturated through attending and presenting lectures: "you somehow learn how to talk," explained an experienced speaker. Seminar presenters pepper their talks with remarks about "what everybody calls" certain objects or citations of "some standard assumptions" and note standard approaches even when not using them. Speakers cite his-

[15] In this sense, the mathematical seminar offers an alternative mode of lecturing to the classical typology of lecturing proposed by Goffman (1981), featuring a form of "fresh talk" that is neither presented nor understood as spontaneous but is simultaneously quite distinct from memorized or strictly rehearsed lecture talk.

torical authorities in relevant subject areas and refer to colleagues (including some present at the seminar) to personalize these allusions. These references to people and concepts work to dissolve temporal as well as professional boundaries. In an interview, one junior researcher spoke undifferentiatedly of insights from a senior colleague gleaned, respectively, from a conversation the previous week and a body of that colleague's work more than two decades old. So, too, do old and new theorems and approaches coexist in a seamless technical matrix on the seminar blackboard, thereby enacting an epistemology of mathematics that actively looks past concepts' context-specificities.

Like the neuroscientists studied by Lynch (1985), subjects for this study organized and narrativized their research activity according to various projects.[16] Subjects typically maintained three active projects concurrently, often with many more investigations "on the shelf." Projects were distinguished by their set of collaborators, their animating questions, and the "tools" or methods they employed. Their progress was marked in researchers' minds by the gradual reification and conquest or circumvention of barriers they classified as conceptual or (less often) technical. Projects rarely end decisively, but can be disrupted by the relocation of a collaborator, stymied or made obsolete by other researchers' results, or stalled before particularly stark conceptual barriers. When a suitable partial result is obtained and researchers are confident in the theoretical soundness of their work, they transition to "writing up". Only then do most of the formalisms associated with official mathematics emerge, often with frustrating difficulty. Every researcher interviewed had stories about conclusions that had either come apart in the attempt to formalize them or been found in error even after the paper had been drafted, submitted, or accepted. Most see writing-up as a process of verification as much as of presentation, even though the mathematical effort of writing-up is viewed as predominantly "technical," and thus implicitly not an obstacle

[16]The project-orientation of labor and narrative seems quite natural for neuroscientific research, with its vast assemblages of researchers, technicians, and apparatus. Given the stereotype of the lone mathematician and the importance of breakthrough stories in post-facto accounts of mathematical innovation, however, the predominance of project-work in mathematics is considerably more surprising.

to the result's ultimate correctness or insightfulness.

Seminars have a special place in the temporal organization of mathematics research. For presenters, presentations can drive the writing-up process by forcing the speaker to cast recent results in a narrative that can be used in both talks and papers, one that mobilizes both program and project to construct an intelligible account of their work (cf. Ochs and Jacoby 1997). Preparing a piece of work for public consumption requires the impartition of an explanatory public logic where ideas develop according to concrete and recognizable methods. Seminars force researchers to articulate their thinking in terms of a series of significant steps, unavoidably changing the thinking in the process by forcing it to conform to a publicly viable model or heuristic. Finally, the seminar's audience joins in the constitution of a shared public logic that frames their own projects in turn.

Thus, the "following" that takes place in the seminar and extends to other areas of mathematical communication consists of more than the mere sequential comprehension of inscriptions and allusions. "Following" structures the production and intelligibility of entire programs of mathematical research, as well as of the communities that engage in those programs. These entities are built along figures of time and topic that underwrite the directed pursuit of new mathematics.

4 An Ostentatious Medium

We have just depicted a seminar room subtly suffused with concepts and allusions, but these invisible entities are largely just a facile shorthand for what takes place in the seminar. Rather than as a trading zone for airy intellections, we aim to treat mathematical communication in terms of the pointings, tappings, rubbings, and writings that more manifestly pervade our subjects' work.[17] In the seminar, these materializations of mathematics are

[17]The observations in this section should be compared to Ochs, Jacoby, and Gonzales's (1994) discourse analysis of physicists' use of "graphic space" to narrate their work and to Suchman and Trigg's (1993) analysis of whiteboard work among artificial intelligence

concentrated around the person of the speaker and the physicality of the blackboard.

There is nothing about the blackboard that is strictly necessary to the mathematician. There are other means of writing equations for personal or public display, other tokens on which to hang one's disciplinary hat. Outside of the seminar room, blackboards play a relatively limited (which is not to say insignificant) role in most mathematicians' daily work. The stereotype of the chalk-encrusted mathematician is nearly as mis-begotten as that of the mathematician lost in his own mental world.

Nevertheless, mathematicians return to the blackboard. Introduced in its present form as a large surface for pedagogic chalk writing near the turn of the nineteenth century, its status as an iconic signifier for the discipline is no accident.[18] Blackboards dominate mathematics in two crucial spheres: the classroom and the seminar. It is with blackboards that young mathematicians learn the ins and outs of their art, and it is on blackboards that established scholars publicly ply their newly-minted innovations. These twin settings enshrine blackboard mathematics as exemplary in a way that pervades all of mathematical practice—whether marked in dust, ink, or electrical circuits—despite the blackboard's ever-growing appearance of obsolescence.[19] It matters little that the full measure of the blackboard's glory is confined to the narrow environs that lend it its profound influence. In the pregnant space between chalk and slate there reposes a germ of the bursts of inspiration, triumphs of logic, and leaps of intuition that dominate mind-centered

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researchers.

[18]We can only note here that blackboards' iconicity is vast. They are ubiquitous props in portraits of theoretical researchers in mathematics and physics—on which, see Barthes (1957, 104–105)—and a widely traded symbol of pedagogic authority and intellectual inspiration, from *Good Will Hunting* to Glenn Beck. On the nineteenth-century pedagogic history of the blackboard, see Kidwell *et al* (2008) and Wylie (2011).

[19]We do not have the space for a systematic discussion of competing technologies to chalk and blackboards, which include alternative writing surfaces as well as tools for projecting text and images. See, however, Barany's (2010, 43–44 *et passim*) discussion of these technologies with reference to adaptations that reinforce the disciplinary centrality of the blackboard even when it is not in use.

accounts of mathematics.[20]

As components of the mathematics department's physical infrastructure, blackboards are most prominent in seminar rooms and lecture theaters. There, multiple boards are typically arranged to span the front of the room, sometimes in sliding columns that allow the speaker to move the boards up or down for writing and display (figure 1). Blackboards are also found in the tea room used by faculty and graduate students and in individual professors' and shared student offices.

Figure 1: An arrangement of sliding blackboards from the Analysis Group's seminar room.

Even as blank slates, blackboards are laden with meaning. As topical surfaces of potential inscription, they define the spatial outlay of lectures and tutorials, guiding audience members in their choice of seats and occasionally demanding that the room be reconfigured to improve the board's visibility.[21] They presage the seminar's rhythm, its steady alternation of marking, talk-

[20]It should be said that blackboards have been made predominantly out of materials other than slate for most of their history. The paradigmatic relationship between blackboard and slate has, however, fundamentally shaped blackboards' social meaning and material development. Nor, for that matter, are blackboards always black. The seminar boards at the heart of this study were a dark shade of green.

[21]Suchman (1990, 315) notes a related phenomenon of whiteboards orienting researchers in a shared interactional space in the more intimate settings of research discussions—a phenomenon we also noted among the mathematicians in our study.

ing, moving, and erasing. They are perpetually at hand: even in conference talks, whose frenetic pace tends to preclude blackboard exposition, they are occasionally mobilized to expand on a point missing from a speaker's prepared slides; in the tea room, conceptual discussions sometimes find their way to the room's otherwise rarely-used boards; in offices, boards serve as notepads for non-mathematical ephemera (such as telephone numbers) in addition to mathematical jottings.

More features appear when blackboards are in use. They are big and available: large expanses of board are visible and markable at each point in a presentation, and even the comparatively small boards in researcher offices are valued for their relative girth. Blackboards are common and co-present—users see blackboard marks in largely the same way at the same time. They are slow and loud: the deliberate tapping and sliding of blackboard writing forces the sequential coordination of depiction and explanation at the board, pacing and focusing speaker and audience alike. They are robust and reliable. And, as noted above, they are ostentatious—so much so that colleagues in shared offices expressed shyness about doing board work when office-mates are present.

As a semiotic technology, the blackboard is as much a stage as a writing surface. That is, boards constitute spaces for mathematical performances that are not reducible to the speaker's chalk writing. Speakers frequently dramatized particular mathematical phenomena, using the board as a prop, setting, or backdrop.[22] Most seminar gestures, however, index rather than indicate mathematical phenomena, exploiting the spatial configuration of the blackboard to organize concepts and settings. That is, rather than indicate

[22]Greiffenhagen (2008, par. 29–66) and Greiffenhagen and Sharrock (2005) make comparable observations for logic instruction. Núñez (2008) offers a contrasting approach to gestures in mathematical performance, seeking fundamental cognitive mechanisms underlying gestures and metaphors used in mathematics. We thank an anonymous referee for pointing out that these gestures, which are either audience-facing or board-facing, take place in a lecture context where nearly all writing is done while the speaker faces away from the audience. Thus, the physical constraints of the board provide a stage that markedly limits the timing and orientation of the gestures available to the speaker at any given point of the lecture.

particular phenomena, the vast majority of observed gestures indicated those phenomena's place in the foregoing exposition—indexing place rather than indicating properties.[23] Proofs are explained with reference to their initial assumptions by pointing at or tapping boards filled with lists of conditions, which are typically placed at the tops of boards even when space remains at the bottom of the board at which the speaker had been writing.

When an argument is invoked for the second time in a lecture, the speaker's hand can trace its earlier manifestation from top to bottom as a substitute for re-reading or re-writing it. A question from the audience frequently prompts the speaker to return a previously-worked sliding board to its position at the time of its working in order to answer queries about the writing thereon, even if no additional marks are made. It is not uncommon to see the speaker's eyes casting about the board for an earlier statement before deciding how to proceed with the next. On multiple occasions, the speaker gestured at a particular statement's former place on the board even after it had been erased, rather than reproduce the statement in another part of the board for the purpose of referring to it.

Specific board locations can carry mathematical significance. Parts of an expression can be separated visually, and corresponding terms are often aligned or written over each other, even when this requires the writer to sacrifice some of the marks' legibility. For instance, when a new bound is introduced for an analytic expression many speakers simply erased the bounded expression and contorted their writing so that the new bound would fit in its place. Similarly, when a proof hinged on the proper grouping or re-grouping of terms in an expression speakers exaggerated the physical spacing between certain terms when writing them. Thus spatialized, statements can be mobilized or demobilized by emphatic or obfuscatory gestures. Multiple speakers, for example, mimed erasing an expression or simply blocked it with

[23] In particular, this observation contrasts with the emphases of Greiffenhagen (2008) and Greiffenhagen and Sharrock (2005). Even in research settings, we found board positioning to be a significant but easily-overlooked instrumental feature of board inscriptions, an observation consonant with Suchman (1990, 315–316).

their hands in order temporarily to exclude it from a consideration or to show that an explanation strategically ignores it.

And what of the marks themselves? One rarely thinks of what *cannot* be written with chalk, a tool that promises the ability to add and remove marks from a board almost at will. The chalk's shape, its lack of a sharp point, and the angle and force with which it must be applied to make an impression, all conspire to make certain kinds of writing impossible or impractical. Small characters and minute details prove difficult, and it is hard to differentiate fonts in chalk text. Board-users thus resort to large (sometimes abbreviated) marks, borrow typewriter conventions such as underlining or overlining, or employ board-specific notations such as "blackboard bold" characters (e.g. $\mathbb{Z}$, $\mathbb{R}$, and $\mathbb{C}$) to denote specific classes of mathematical objects.

Not every trouble has a work-around. Similar to a ball-point pen or pencil on paper, chalk must be dragged along the board's surface to leave a trace. Entrenched mathematical conventions from the era of fountain pens, such as "dotting" a letter to indicate a function's derivative, stymie even experienced lecturers by forcing them to choose between a recognizable dotting gesture and the comparatively cumbersome strokes necessary to leave a visible dot on the board.

The consequences of chalk for mathematics are not just practical but ontological and epistemological. As Livingston (1986, 171) observes, mathematical proofs are not reducible to their stable records. Arguments are enacted and validated through their performative unfolding—an unfolding as absent from circulable mathematical texts as it is essential to the production and intelligibility of their arguments. Like the proofs it conveys, blackboard writing travels only through rewriting. Unlike the marks in books, papers, or slides, blackboard inscriptions can only ever unfold at the pace of chalk sliding against slate. The intrinsic necessity of bit-by-bit unfolding in mathematical exposition is thus built into chalk as its means of writing.

This unfolding matches the "following" mode discussed above, and extends to the audience's listening practices. Few audience members took notes during the seminar. Most who did made only an occasional jotting of a theorem or reference to pursue afterwards. But those who did take extensive notes endeavored to make a near-exact transcript of what the speaker wrote on the board, reproducing a routine practice from their early mathematics coursework and training. The expectation of transcription obliges the speaker to make the board's text self-contained and accountable, leading to a striking duplication of effort between writing and speech whose epitome is the stereotypical speaker who reads his talk off the board as he writes it. The practice of "following" thus impinges both on the global narrative of the talk and on the textual sub-narrative confined to the speaker's marks on the board.

The mutability of blackboard writing, moreover, enacts a specifically Platonist ontology of mathematics. In this view, mathematical objects and systems have an independent existence that is separate from their descriptions, and the same entity can be described in a variety of ways. On a blackboard, lecturers frequently amend statements and definitions about these mathematical entities as their specific properties and constraints are made relevant by the exposition or by audience interrogation. On such a medium, the fact that the once-written text does not tell the final story about a mathematical concept allows a potentially infinite variety of descriptions simultaneously to apply to an object or situation under consideration. Where Suchman (1990, 315) and Suchman and Trigg (1993, 160) depict the board as the medium for making objects concrete, we would stress the board's corresponding ability to make those concepts mutable without threatening their persistence as Platonic entities. Thus, when a speaker returns later to add a necessary condition to a definition or theorem-statement, it can be seen as an omission rather than an error in the speaker's argument—the condition can be made to have been there all along at any such point as that anteriorized conceptual vestment is required for the lecture to go forward.

The logic of blackboard writing governs mis-statements as well as omissions. When the speaker reconsidered a statement and deemed it false, the offending marks could be rubbed out without incident, preserving the veracity of the blackboard record. The dusty traces of the statement's removal cue those few taking notes by pen or pencil in the audience as to which items have been removed so they can appropriately modify their own transcripts. In other situations, a statement was not necessarily false but, usually after an audience enquiry, was judged to be either misleading or beyond the scope of the presentation. In these cases, the speaker could cross out the statement, removing it from the accountable portion of the talk but preserving it among the lecture's mathematical residues.

The availability of different modes of erasure also has narrative consequences. Minor corrections can be made using the side of one's hand to erase small areas of the board while producing an audible thud that preserves the ongoing sequence of words and board-sounds in the speaker's story. Larger erasures, however, must be made with a separate instrument whose use requires the interruption of such discursive sequences—a desirable effect at the end of a planned segment of a talk and an appropriate one where the speaker must "reset" an argument after a significant lapse. The narrative break of clearing a board establishes a board-sequence division that holds even when a new board is available. Before embarking on a new part of an argument, speakers sometimes clear multiple boards to avoid having to erase one mid-sequence. Conversely, if a narrative sequence overruns its allotted board space the speaker sometimes squeezes the remaining text in blanks on the current board rather than move to a new one, often at the cost of legibility.

A final point weds the ontological, epistemological, and practical significance of blackboards. In seminars and offices alike, blackboards are used and experienced as places for translating complex, symbol-intensive ideas into a manipulable, surveyable form. Figure 2 shows an office board that had been used to work out a complicated expression from a published paper. The board shows evidence of insertion, annotation, and erasure. At the top, the researcher started to frame his ensuing writing by singling out the expression from the paper he aimed to comprehend, labeling it with "To show." The expression of interest, the researcher realized in the midst of copying it out, was not so far removed from the chain of reasoning used to demonstrate it, so he moved to the center of the board and wrote (in appropriate shorthand) the entire chain of reasoning. Here, as he described it, the challenge was not grasping a particularly complex series of manipulations but rather understanding a complicated array of indices as a whole.

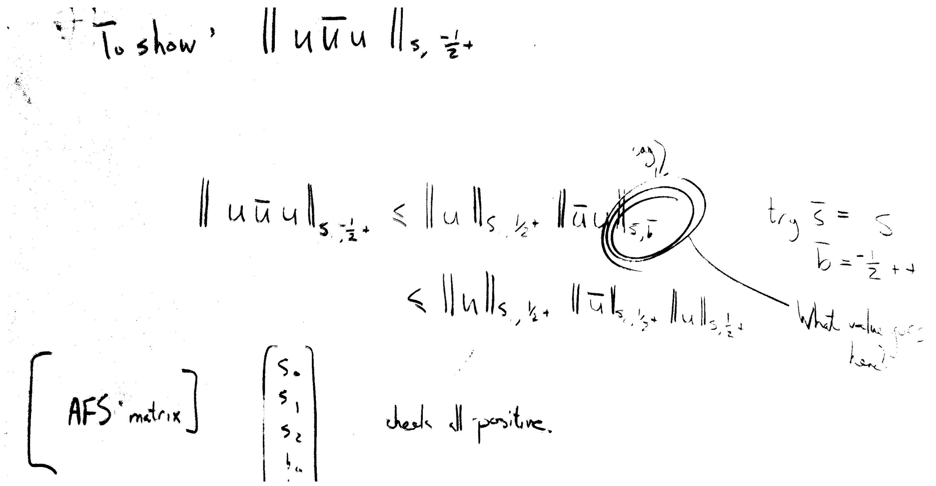

Figure 2: A example of blackboard work from a respondent's office (colors digitally inverted and contrasted).

After copying what he identified as the relevant expressions from the paper, the researcher proceeded to annotate it in terms of questions that would need to be satisfied for the chain of reasoning to be valid and techniques that could answer those questions. On the right of the expression, the researcher attempts to understand the expression by specifying more features of the calculation than are present in the more general form in the paper, qualifying this specification with the note "say." Through this blackboard work, a supposedly abstract datum of certified knowledge becomes a self-identical yet pliable chalk instantiation. We were told that only in this latter form could the researcher comprehend and hope to use that expression, and yet that very form and all its advantages were stuck, for all practical purposes, on the board.

5 Proofs and Reformulations

A dominant theme in sociological accounts of laboratory sciences is the remarkable amount of labor and machinery—in Lynch's (1990, 182) formulation, taken-for-granted "preparatory practices"—devoted to producing texts which can materialize and stabilize unruly natural phenomena in the form of data, plots, and other representations—what Latour (e.g. 1990) called "immutable mobiles." Mathematicians face, in a sense, the opposite problem: the phenomena they study are not unruly enough. Mathematicians thus spend remarkable amounts of labor to materialize their objects of study, but with the goal of coaxing those objects to behave in some new way, rather than to hold some stable and circulable form.

There are thus two fundamentally different kinds of mathematical texts. There are papers and reports akin to journal articles in the natural sciences, but there are also tentative, transitory marks that try to produce new orders out of old ones (with a crucial stage of disorder in between). Blackboards, we have suggested, are the iconic site of this second sort of text-making. Like the natural phenomena scientists try to tame, blackboard writing does not move well. On the other hand, blackboard writing seems supremely open to annotation, adaptation, and reconfiguration. Symbols and images can be erased, redrawn, layered, counterposed, and "worked out" on the board's surface. Such "immobilized mutables" form a constitutive matrix for mathematical creativity.

This "blackboard" way of working with texts is not, therefore, limited specifically to board writing. Asked while away from his office to describe his work space, one interviewee began with the piles and piles of paper covering his desk (figure 3). Populating those piles are reprints of articles, teaching notes, and, most importantly, page after page of scrap paper. The inscriptions of mathematical research, while implicating blackboards, whiteboards, computers, and other media, seem mostly to subsist in the sort of notes that suffuse the spare sheets of paper from our respondent's desk.

Scrap paper writing shares many characteristics with chalk writing. Both

Figure 3: One respondent's paper-covered desk.

rely on augmentations, annotations, and elisions as concepts are developed through iterated inscriptions designed to disrupt the formal stability of mathematical objects. Such iterated efforts at proving, most of which are seen as unsuccessful, produce a long paper trail.[24] One would expect this scrap paper trail, at least, to be somewhat more mobile than blackboards. Not so: for the purposes of research, the process of writing appears to matter more than the record it produces. Scrap paper is almost never mobilized beyond its initial use. One respondent explained that "I don't tend to look back very much." Another has a policy of saving notes until he no longer recognizes the calculations, but confesses that he too rarely looks back at them. "I do a lot of stuff in my head," a third researcher recounted, and his research notes reflected this self-conception by rarely travelling beyond the sites in which they were produced.

Merz and Knorr Cetina (1997, 87, 93) describe mathematical work as a process of "deconstruction," where equations from problems are successively transformed through a variety of techniques until they yield a new theoretical insight. One of our subjects described the process perfectly:

> I'm going to keep doing the calculations again, only now trying to

[24]Latour (1990, 52) identifies the production and legitimation of such cascades of inscriptions as a decisive puzzle for the anthropology of mathematics.

look for terms of this form. . . . I have an ocean of terms like this, and the problem in some sense is how do you put them together so that they make some sense.

Consider how terms are put together in the research notes excerpted in figure 4. This researcher's deconstruction begins with the operator L, whose effect on a function u is first written compactly on the left-hand side of the equation in his notes. (The brackets identifying this expression as Lu were added after the fact as he explained this inscription during an interview.) On the right there appears a nearly-identical expression, with a space opened up between the ∂_j and the rest of the expression's summand (that is, $a_{jk}(x)\partial_k u$). Brackets beneath the two sets of symbols identify them as members of specific families of mathematical objects, respectively $S^1_{1,0}$ and $L^\infty S^1_1$, and the latter identification merits a written-out speculation about a technique ("symbol smoothing") and a desired outcome (inversion). All the while, these textual tokens are experienced and described as ideas. In this example:

> We have some variable coefficient operator [L] that looks like the Laplacian, and so . . . [we] split it up into a sum of pieces, I guess a product of two things. In my case, the first product . . . is just a derivative, and . . . the second factor, less is known about.

$$\underbrace{\sum_{jk} \partial_j a_{jk}(x)\partial_k u}_{Lu} = \sum \underbrace{\partial_j}_{\in S^1_{1,0}} \underbrace{a_{jk}(x)\partial_k u}_{L^\infty S^1_1}$$

probably ~~need~~ symbol smoothing; not clear how to invert.

Figure 4: An equation from an interviewee's research notes.

In addition to regrouping, symbols can be transformed according to mathematical principles and with the help of auxiliary equations and images. Notations and framings are often adapted to particular approaches. "There's a lot of notation, and it does help to go back and forth between them," offered one researcher. Moving between different variables and expressions can coax a troublesome formulation to resemble a familiar one or allow researchers to break a problem into smaller parts. Annotations can also declare aspects of a problem to be difficult, promising, or solved. In one particularly dramatic example of this, an interviewee recounted how

> I put that in a red box because I was very excited when I realized that.... In my mind it moved us closer to completion of the project.

As concepts are continually re-materialized, salient details are expanded or omitted, much as they would be on the blackboard. One researcher's notes had the word "factor" in place of a positive constant whose particular value was not relevant at that stage of the investigation. He expected that he might ultimately "see sort of which ones [factors] are ones that are helping you prove your result and which ones are the obstacles," and could then manage the obstacles separately. The process described by multiple respondents involved successive attempts to develop and refine a proof, with each attempt aimed at managing a new set of constraints after one is convinced of the proof's "main idea."

One should not get the impression, however, that the only papers of significance in a mathematician's office are scrap papers. A large amount of space is devoted to storing books and articles that contain mathematics in its most stable and circulable form. These are achieved through the "writing up" process, which (in our subjects' consensus) takes place strictly after the genuinely creative part of mathematical research—though all admitted that the form and often the substance of a result were liable to change substantially during or even after writing up. The work of writing up deserves a separate study—parts of it are addressed lucidly by Rosental (2008) and Merz and Knorr Cetina (1997). For our purposes, we would like to expand upon the inverse phenomenon: the less-recognized reading practices that convert "written-up" prose into a form usable in mathematical research—practices that might be called "reading down."

There is a crucial difference between mathematical papers and reports of scientific experiments. Where the latter are understood to depend on the credible reporting of experimental outcomes, the former are seen in principle to contain all the apparatus of their verification. That is, where scientists must describe experiments and plot data, mathematicians are expected to reproduce in meticulous detail each of the novel rational steps behind their conclusions. The time and thought required to understand and verify each such detail makes mathematical papers subject to similar issues of trust, credibility, and reproducibility as have been described for the natural sciences, but the presentation of mathematical texts as (in principle) self-contained means that their circulation and deployment can have a decidedly different character.

In particular, mathematical texts present readers with two kinds of useable information. They establish lemmas and theorems that can be invoked as settled relations between specific mathematical phenomena, and they present methods and manipulations that can be used by others to establish different results. Researchers access others' papers through preprint and citation databases, and in smaller specialisms researchers will simply send preprints to a regular list of colleagues. They approach their stream of available papers using successive filters to identify where the two foregoing types of information most relevant to their research will be found. The process of perusing a database, for instance, might start with reading the titles of articles in relevant subject areas, the abstracts of articles with relevant titles, and so forth. The mathematicians with whom we spoke almost never read papers in their entirety—and certainly not with the goal of total comprehension.

When information from a paper is deemed immediately relevant to an ongoing project, it is finally read for its technical detail. Rather than attempt to digest every claim, however, readers try to identify concepts, formulations, and conclusions that are recognizable in the context of their own work. These identifications begin a process of re-rendering papers' key passages in terms readers hope may ultimately advance their instrumental research goals.

This process of reformulating official papers into research instruments can span several media. A single page of one researcher's notepad, shared during an interview, visibly manifested a series of translations from an article to penned equations to an email to spatial gestures and further writings. Interviewees reported experiencing mathematical concepts in terms of formalisms, properties, or operations. One described an equation by placing invisible terms in the air, one by one, in front of him. "I've written it down so many times," he explained, that he instinctively saw "the first order terms appear here and the second order terms there." Another used a box of tea on his desk as an impromptu prop in explaining a source of theoretical consternation from a recent effort. Different modes of mathematical cognition must necessarily interact to produce the transformations that bring about original proofs and theorems—transformations that would not generally be possible within a single framework of representation. Moreover, they must interact in a way that allows the coordination of mathematical understanding between different researchers in a variety of settings.

This leaves mathematical ideas in a strange position. Particular and ideosyncratic inscriptions and realizations are utterly central to the practice of mathematics. Paradoxically, mathematical inscriptions (especially on blackboards) work in ways that specifically (and, as we have argued, misleadingly) assert the opposite—that ideas somehow do not depend on the ways in which they are mobilized. The flexibility of mathematical representations obscures the socio-material coordination necessary to move concepts so freely from one form to another. Mathematical work rests on self-effacing technologies of representation that seem to succeed in removing themselves entirely from the picture at the decisive junctures of mathematical understanding. It is only by virtue of these disappearing media that one can be said to understand a concept itself rather than its particular manifestation.

Except when one cannot. Like scientific instruments, mathematical representations are subject to "troubles," flaws, and shortcomings (see Lynch

1985). The vast majority of attempts to use material proxies in one form or another to elucidate a concept are not counted as successes within a program of research. Seminars are among the rare displays of mathematical semiosis in a research setting where it is understood and expected that the signs will work. Mathematical research is marked by the constant struggle to create viable signs. As one of our subjects put it:

> It's largely having a model and trying to get the new thing to fit into the old model, and at certain points that simply fails, and at that point you sort of mess around and think about ... the old one a slightly different way, sometimes just calculating [and] seeing what comes out ....

6 Representation in Mathematical Practice

In most people's experience, mathematics is a static body of knowledge consisting of concepts and techniques that are the same now as they were when they were developed hundreds or thousands of years ago and are the same everywhere for their users and non-users alike. Little would suggest that there are corners of mathematics that are changing all the time, where as-yet unthinkable entities interact in a primordial soup of practices that constantly struggle to assert their intelligibility. Such is the realm and such are the objects of mathematical research.

The relationship between mathematicians and their objects of study is anything but straightforward. There is no mathematical concept whose formal immediacy or self-evidence stands beyond media and mediation. As a science of ideals, mathematics rests on the capacity of mathematicians to legitimize and manipulate particular representations of mathematical phenomena in order to elucidate rigorous mathematical knowledge.

In contrast to well-worn accounts of representation in the natural sciences, the story of mathematics is less about the hidden work of taming a natural phenomenon according to ideals than about the very public work of crafting those putatively independent ideals from their always-already-dispensable material manifestations. We have proposed chalk as both a literal and a figurative embodiment of that work. As a physical means of representation, chalk and blackboards entail a potent but highly circumscribed means of publicly materializing mathematical concepts. Their mode of representation, moreover, defines and influences mathematical practices far beyond those relatively limited circumstances where the mathematician actually has chalk in hand.

Mathematical writing and the mathematical thinking that goes with it are markedly dependent on the media available to the mathematician. Mathematical work traces the contours of its surfaces—there is little that is thinkable in mathematics that need not also be writeable, particularly in the mathematics that is shared between mathematicians.[25] Blackboards, paper, and other media make certain forms of writing, and hence certain kinds of arguments and approaches, more feasible than others. Without having to assert that the limits on mathematical inscription definitively foreclose many potential truths from *ever* being described and accepted mathematically, it is manifestly clear that those limits *can* and *do* imply corresponding constraints on the lived and daily course of mathematical research. As De Millo, Lipton, and Perlis (1979, 274–275) put it, "...propositions that require five blackboards or a roll of paper towels to sketch—these are unlikely ever to be assimilated into the body of mathematics."

Even when a viable constellation of representations is found, the mathematician's work is not done. These multifarious semiotic entities must then be made accountable to the equations, syllogisms, and arguments found in the published literature that compose the official corpus of mathematical knowledge—a project for which they are poorly adapted. A staggering portion of mathematicians' work goes into decoding published papers to create functional intuitions and understandings and, conversely, into encoding

[25] Rotman (1993, x) likewise asserts an interweaving of thought and inscription, though his focus is on the semiotics of mathematical abstractions rather than the paticularities of mathematical research and communication.

those intuitions in the accountable forms in which they will be credited as genuine. This is why chalk and seminars are so important. They give researchers shared partial access to what is so obviously missing from official accounts of completed work: namely, the experienced material performance of mathematics in action. The tension between circulation and application in mathematics is a real one. Mathematical ideas are not pre-given as the universal entities they typically appear to be. The most important features of mathematics can be as ephemeral as dust on a blackboard.

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